A robust framework for the generation of random metamaterials based on a graph algorithm

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ABSTRACT

In the realm of metamaterial research, the exploration of random structures presents an innovative path less traveled, compared to the conventional focus on periodic designs. Our study introduces a novel framework for generating random metamaterials using graph algorithms, which ensures connectivity and adaptability across a multitude of base shapes, such as cylinders, triangles, pyramids, and cubes. This flexibility enables the application of our designs across various domains, allowing for the investigation of properties including stiffness, density, and acoustic impedance. By leveraging graph algorithms in our framework, data representation and manipulation become more intuitive and efficient, facilitating the design process. Our approach demonstrates significant versatility in manipulating the macroscale and microscale elements of the designs, providing a tailored fit for specific applications. We present a series of designs, showcasing the ability to control and predict the material's behavior under different conditions. The designs can be effectively implemented across various fields and subjected to multiple analytical studies, encompassing static, dynamic, and eigenfrequency assessments. Properties such as impedance, stiffness, density, and more can be explored, opening the door to a wide array of applications and potential innovations in metamaterial research. We illustrate the computational results for stiffness and acoustic impedance, highlighting the method's efficacy through examples ranging from rod-based to cube-based designs. This framework not only paves the way for advancements in metamaterial research but also opens up new possibilities for innovation in fields requiring customized material properties.

Keywords: Metamaterial, acoustic, design, random, graph algorithm

1. INTRODUCTION

Metamaterials are artificially engineered materials that exhibit extraordinary properties beyond those found in naturally occurring substances.[1,2] These materials are capable of manipulating light, sound, and other physical phenomena in ways that conventional materials cannot achieve.[3] Acoustic metamaterials, for example, can control acoustic waves by blocking, absorbing, enhancing, or bending them.[4] They are used in various applications, including noise reduction in buildings,[5] ultrasonic imaging in medicine,[6] vibration control in aerospace,[7] sound enhancement in audio devices,[8] and seismic wave mitigation.[9] Incorporating randomness into metamaterial design has gained interest, inspired by the efficient yet disordered structures observed in nature, such as the cell organization of carnivorous plants, the crossed-lamellar textures of shells, and the hierarchical organization of trabecular bone.[10] This approach has led to enhanced robustness against manufacturing defects, improved stiffness-to-weight ratios, higher failure resistance, and controllable negative Poisson's ratios.[11]

There are several approaches to generate metamaterials, including topology optimization,[12] inverse design methods,[13] and machine learning algorithms.[14] Incorporating artificial intelligence in designing structures like

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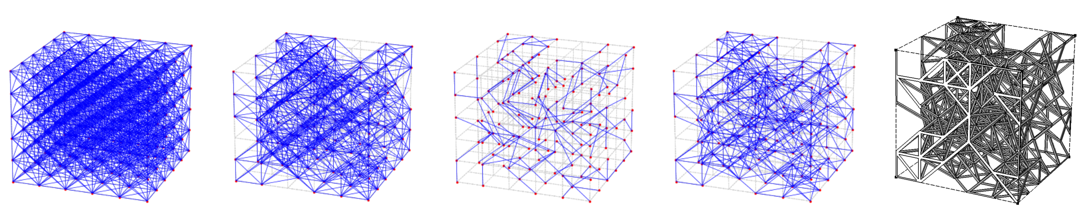
metamaterials presents significant challenges, particularly in building a substantial library of models, which is a time-consuming process. To address this, researchers are exploring more efficient ways to represent data and geometry. The use of graph-based topology representations, for example, allows for the application of machine learning techniques to identify the relationships between structure and properties.[15] Several studies have explored graph-based approaches to enhance the process of designing metamaterials. For instance, a procedural graph is used to streamline the modeling process of symmetric unit cells by connecting various operations within a graph framework.[16] Primal and dual graphs are employed to systematically construct shell-lattices, providing a versatile framework for designing lattice structures with desired properties.[15] Additionally, graph theory has been applied to design and analyze origami-based mechanical metamaterials, where unit cells are represented as vertices and their connections as edges to efficiently explore geometrical configurations.[17]

Existing representations for structural designs, such as voxel grids, are versatile but cumbersome when it comes to representing and editing diverse structures like beams, thin shells, and solid bulks.[16] These limitations underscore the need for more efficient generation and representation methods. In this paper, we propose a general graph-based method for generating random metamaterials to address these challenges. This approach is applicable to a variety of base shapes, including triangular shells, pyramids, and cubes, in addition to rod-based metamaterials. By leveraging graph theory, specifically spanning trees, our method ensures interconnectivity within the designs and overcomes the difficulties associated with generating disordered metamaterials. This method streamlines data representation and manipulation, facilitating complex design processes. The flexibility and efficiency of this approach demonstrate its promise across different fields due to its ability to generate designs rapidly and efficiently.

2. METHODOLOGY

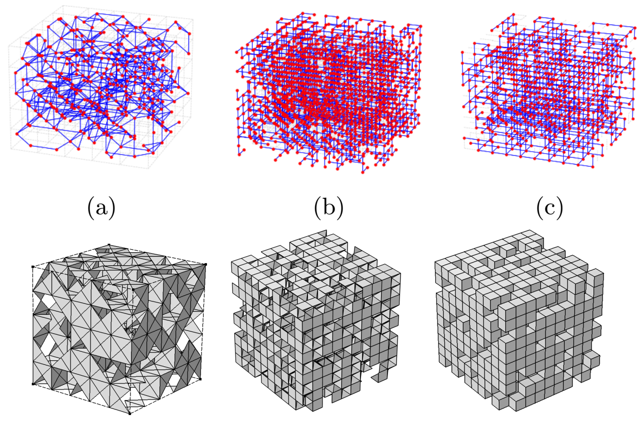
In this section, we present our novel method for generating metamaterial structures applicable across various geometries. We begin by defining the design domain as a large gridded cube where dimensions and vertex count can be adjusted to suit specific requirements. Vertices are connected through edges, forming a fully connected graph (see Fig. 1a). This graph serves as the design domain from which the metamaterial design is selected. Next, we randomly select a percentage (p_1) of the vertices to form a fully connected subgraph (Fig. 1b). Despite the random selection, vertices maintain connections with all available adjacent vertices. A spanning tree is then generated from the subgraph (Fig. 1c) to ensure connectivity in the final design by selecting critical edges necessary for interconnectivity. To avoid a sparse design, additional edges are introduced into the spanning tree graph (Fig. 1d). This step involves subtracting the edges present in both the fully connected subgraph and the spanning tree graph to form an admissible edge set. Finally, a percentage (p_2) of these admissible edges is randomly selected and added to the spanning tree graph. The resulting metamaterial design is then mapped into a rod-based configuration, where each graph edge corresponds to a rod, bar, beam, or cylinder in the metamaterial design (Fig. 1e).



(a) (b) (c) (d) (e)

Figure 1: Procedure for generating rod-based metamaterials using graphs. (a) Fully connected graph formed by edges. (b) Fully connected subgraph formed by randomly selecting a portion of vertices. (c) Spanning tree ensuring connectivity. (d) Densified subgraph with additional edges. (e) Rod-based metamaterial design corresponding to the densified graph.

Following a similar methodology, we define other graphs to generate designs for metamaterials using triangular shells, pyramids, and cubes, as shown in Fig. 2. These designs can be seamlessly transferred to various software or code packages for analysis in various applications.



(a) (b) (c)

(d) (e) (f)

Figure 2: Graphs used to generate metamaterials using (a) triangular shells, (b) pyramids, and (c) cubes. Corresponding metamaterial designs using (d) triangular shells, (e) pyramids, and (f) cubes.

3. RESULTS AND DISCUSSION

In this section, we present several designs generated by our algorithm and analyzed using COMSOL Multiphysics®. It is worth noting that the algorithm's output can be imported into various commercial software or code packages for further analysis. These designs have versatile applications across multiple fields. This study emphasizes the properties of stiffness and acoustic impedance as exemplars.

To illustrate the capabilities of the generation method, we created designs incorporating rods, triangular shells, pyramids, and cubes, as depicted in Fig. 3. The stiffness and acoustic impedance were calculated in three principal directions, with results presented in Figs. 4. The results indicate that the properties of the designs are consistent across the three principal axes, reflecting the inherent randomness and uniformity achieved through our method.

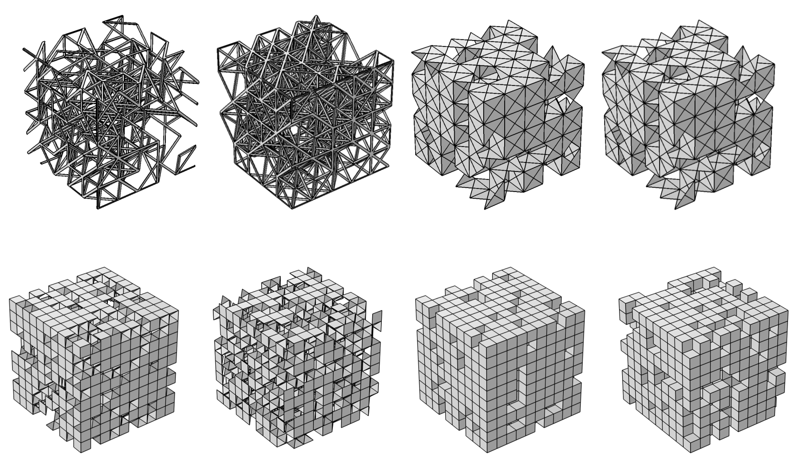
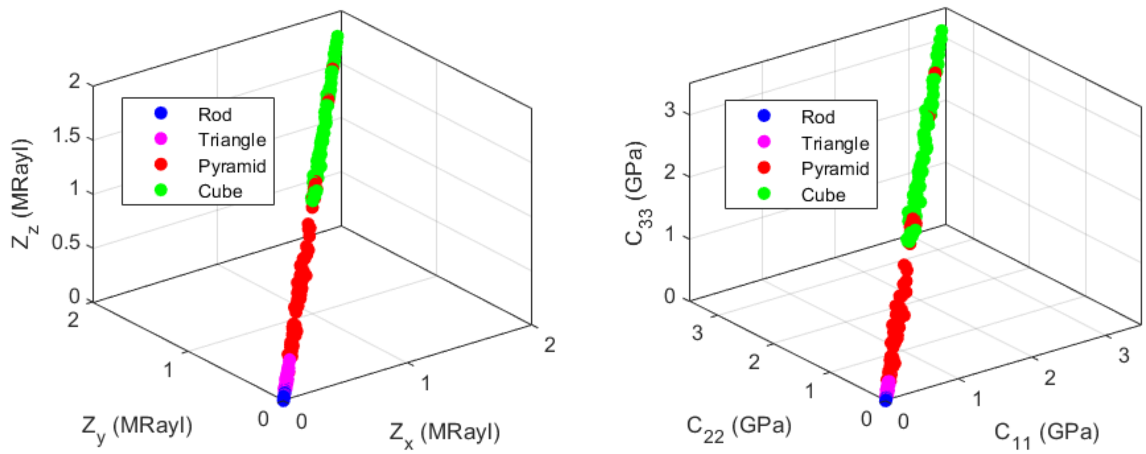


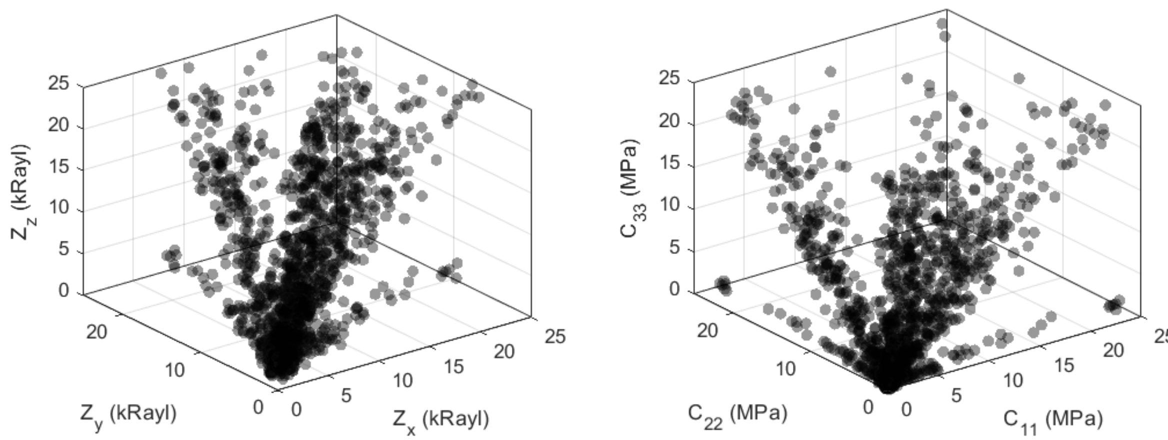
Figure 3: Schematic representation of various designs generated by the framework, illustrating configurations based on rods, triangular shells, pyramids, and cubes.

An additional notable feature of the method is its ability to bias the behavior toward a specific direction. After forming the spanning tree, we can densify the design by adding more edges along our desired direction. This strengthens the design in that direction and expands the property space covered by our designs. Fig.5 shows rod-based designs that are strengthened along various directions.



(a) (b)

Figure 4: Calculated (a)impedance and (b) stiffness for the designs along three principal directions.



(a) (b)

Figure 5: Biasing (a) the impedance and (b) stiffness of the rod-based designs toward various directions.

4. CONCLUSION

In summary, we proposed an original and efficient method for generating random yet connected metamaterials that can serve different purposes and can be used in various applications. The method leverages graph algorithms to visually represent complex data and expedite the generation process. It also benefits from spanning trees to ensure interconnectivity within the designs. Moreover, the method is capable of generating designs based on several fundamental shapes. Here, we demonstrated the designs generated by four base shapes which were rods, triangles, pyramids, and cubes. We also explored the two key properties of the designs which were stiffness and acoustic impedance. We showed that the designs exhibit consistent behavior along three principal axes. Furthermore, we extended the method to introduce a bias toward a desired axis for rod-based designs, enhancing their strength in a specified direction. This feature is particularly beneficial for covering a large property space using a single base shape.

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